Game Theory and Olympic Tae Kwon Do

The sport of Tae Kwon Do, differs form the traditional martial art in many ways. The first and most noticeable is the sport is not used to injure or incapacitate an opponent where this is part of the martial art. Much like the difference between sword fighting and fencing. When Tae Kwon Do was inducted in to the Olympic Games a few more changes were made to create a unique event. To keep it different for Boxing punching no longer scored any points (this rule has recently been revised) and punching to the face is disallowed, (mainly for safety reasons as TKD player do not where padding on their hands). To avoid similarities to Judo and Wrestling grappling and tripping techniques were also removed. Thus all that was left was kicking, blocking and other defensive maneuvers. The sparring competition is won on a point basis. The basic rules of Olympic tae kwon do sparing are;

Kicks to the body are 1 point
Kicks to the sides or top of the head are two points
Kicks to the face (intentional) are; a warning for the first offence, - 1 point the second time, and disqualification the third offence.
Punching to the face (intentional) immediate disqualification
Punching to the body 1 point for knock down or out of the ring otherwise no points
Leaving the ring; warning first time, -.5 point second offence, -full point on the third time.
Falling down; warning first time, -.5 point second offence, -full point on the third time.
In a match one player wears a blue chest pad and the other a red chest protector for identification purposes.

First we will look at a simplified (one action) zero sum model of a match. We will assume that the players are evenly matched. That is any time an attack is defended it is unsuccessful. (Ties are represented with a 0, if both players gain the same number of points there is no advantage)

		Blue	
		Attack	Defend
Red	Attack	0	0
	Defend	0	0

This pay off matrix is very boring and of little help. In order to gain more insight we will try expanding the attack entry in to the two different point values for kicking to the body or the head.

Now we have the payoff matrix:

		Blue		
		defend	Head	body
	Defend	0	0	0
Red	Head	0	0	1
	Body	0	-1	0

With the minor adjustment of splitting the attack into two categories the matrix has become slightly more useful. However after looking at domination we end up with a strange result.

		Blue		
		defend	head	body
	Defend	0	0	0
Red	Head	0	0	1
	Body	0	-1	0

(Dominated rows are marked off in red and the columns in blue)

It appears that the red player should only concern him/herself with kicking the blue players head and the blue player should chose between head attacks or defense. At first it may seem that this is unrealistic, nevertheless in real matches this strategy can be observed especially in the case of skilled black belts whom fell they have the speed necessary to keep their opponents on the defensive. Even though this is seen in matches it is not the only way the game is played.

Next we will make additions to the matrix, that represent more complex maneuvers and strategies. The first of these is the back out or fall. Recall that both of these events have the point successive point penalty of: warning, -.5, -1. For ease of calculation we will assume a half point penalty for each offence. The reason for adding this strategy is a player can opt to take the half point but be out of the scoring range of her/his opponent and thus not lose a full point form the attack.

		Blue			
		defend	head	body	OB or Fall
	defend	0	0	0	0.5
Red	head	0	0	1	0.5
	body	0	-1	0	0.5
	OB or Fall	-0.5	-0.5	-0.5	0

Taking out dominated rows and columns gives us.

		Blue			
		Defend	Head	body	OB or Fall
	defend	0	0	0	0.5
Red	head	0	0	1	0.5
	body	0	-1	0	0.5
	OB or Fall	-0.5	-0.5	-0.5	0

Again we are left with the same strategies as before.

So we will now look at a more complex version of the fall, one in witch a player falls (taking the half point penalty) in order to avoid an attack while attacking him/herself.

		defend	head	body	Fall kick H
	defend	0	0	0	0.5
Red	head	0	0	1	-1.5
	body	0	-1	0	-1.5
	Fall Kick H	-0.5	1.5	1.5	0

After sorting through the dominated strategies we are left with no match at all.

			Blue		
		defend	head	body	Fall kick H
	defend	0	0	0	0.5
Red	head	0	0	1	-1.5
	body	0	-1	0	-1.5
	Fall Kick H	-0.5	1.5	1.5	0

Neither player does anything it is just a match of waiting to block an attack that will never come.

If we put both types of out of bounds falling techniques in to the payoff matrix gives us a bit more to work with.

			Blue			
		defend	head	body	OB or Fall	Fall kick H
	Defend	0	0	0	0.5	0.5
Red	Head	0	0	1	0.5	-1.5
	Body	0	-1	0	0.5	-1.5
	OB or Fall	-0.5	-0.5	-0.5	0	0
	Fall Kick H	-0.5	1.5	1.5	0	0

After removing dominations we have

		Blue				
		defend	head	body	OB or Fall	Fall kick H
	Defend	0	0	0	0.5	0.5
Red	Head	0	0	1	0.5	-1.5
	Body	0	-1	0	0.5	-1.5
	OB or Fall	-0.5	-0.5	-0.5	0	0
	Fall Kick H	-0.5	1.5	1.5	0	0

A.k.a.

		Blue	
		Defend	Fall Kick H
	Defend	0	0.5
Red	Head	0	-1.5
	Fall kick H	-0.5	0

At this point we will look for mixed strategies.

Let W={w1,w2,w3} be the percents of Red's strategies

$$\{w1, w2, w3\}.\begin{pmatrix} 0 & 0.5` \\ 0 & -1.5` \\ -0.5` & 0 \end{pmatrix}$$

{-0.5 w3,0.5 w1-1.5 w2}

This implies w3=0 (this reflect the a row was "missed" in the domination elimination)

$$\{w1, (1-w1), 0\}.\begin{pmatrix} 0 & 0.5` \\ 0 & -1.5` \\ -0.5` & 0 \end{pmatrix}$$

{0,-1.5 (1-w1)+0.5 w1}

Solving 2w1-1.5=0 gives us w1=.75, w2=.25

W2={.75,.25,0}
{0.75,0.25,0}

Mixed strategies for Blue

let K={{k1},{k2}} be the percents of blue's strategies

$$\begin{pmatrix} 0 & 0.5` \\ 0 & -1.5` \\ -0.5` & 0 \end{pmatrix}.\begin{pmatrix} k1 \\ 1-k1 \end{pmatrix}$$

$$\begin{pmatrix} 0.5\,(1-k1) \\ -1.5\,(1-k1) \\ -0.5\,k1 \end{pmatrix}$$

Solving 1.5k1-1.5=-.5k gives us k1=.75 k2=.25

If we put these mixed strategy values into the matrices to find an expected value we find.

$$(.75 \quad .25) . \begin{pmatrix} 0 & .5 \\ 0 & -1.5 \end{pmatrix} . \begin{pmatrix} .75 \\ .25 \end{pmatrix}$$

{{0.}}

This really should not be surprising since we assumed that both players were equally matched.

The question of adding more and more complex maneuvers to the payoff matrix needs to be addressed, or in other words when do we accept that we are in a losing battle. In the next generation of matrix we have added the “Combo”. In this maneuver the player not only blocks the opponents kick but punches to the chest at the same time (to gain distance) and though a counter kick. This is a complex maneuver the player whom whishes to do this has to swing one arm down to intercept an incoming kick while punching with the other arm. Mean while her/his legs are performing skilled dance of their own. Firstly the front and back legs switch places in a triangle form. Now the new back leg initiates a kick to the opponents newly exposed body. Recall all this is going on while the arms are doing their “thing”. But even this matrix breaks down in to both players performing this new better maneuver.

				Blue			
		Block	Head	Body	OB/Fall	Fall Kick h	Combo
	Block	0	0	0	0.5	0.5	-1
	Head	0	0	1	0.5	-1.5	-1
Red	Body	0	-1	0	0.5	-1.5	-1
	OB/Fall	-0.5	-0.5	-0.5	0	0	-0.5
	Fall Kick h	-0.5	1.5	1.5	0	0	na
	combo	1	1	1	0.5	na	0

(not applicable is entered since this combo is only performed against an attack)

It certainly seems that the payoff matrix can continually added on to with newer and better maneuvers, but we will always end up in a situation of a draw. Thus it is time for us to admit that a zero sum approach is not the way to solve this game. This makes sense because the match is won by the player with the most points not by a balance of points.

In future applications of game theory to sparing, possible avenues to explore would be using a non-zero sum matrix. Or by expressing the different skill levels of the two players in some fashion, perhaps as a percentage of successful attacks and blocks. The use of a utility function seems very promising. With such a function not only could we account for different skill sets in the players but also the different difficulties of the maneuvers themselves.

			Player 2	
		defend	head	body
	defend	0,0	0,0	0,0
Player 1	head	0,0	2,2	2,1
	body	0,0	1,2	1,1